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VERTEX-TO-VERTEX MEDIAN AND VERTEX-TO-EDGE MEDIAN OF A DOUBLE LOLLIPOP

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Abstract

In this paper, we describe some definitions and results on the vertex-to-vertex medians and the vertex-to-edge medians of general graphs and some particular graphs. Then we introduce the definition of a double lollipop which is a particular type of a bicyclic graph and investigate the structures of the vertex-to-vertex median and the vertex-to-edge median of a double lollipop.

Keywords: vertex-to-vertex median, vertex-to-edge median, bicyclic graph, double lollipop.

1. Some Graph Theoretic Terms and Notations

We first introduce some graph theoretic terms and notations which are used in this paper.

A ***graph*** $G=(V(G),E(G))$ consists of a nonempty finite set $V(G)$ of ***vertices*** and a finite set $E(G)$ of ***edges*** where $E(G)$ is disjoint from $V(G)$ and each edge of $E(G)$ corresponds to an unordered pair of (not necessarily distinct) vertices of $V(G)$. If an edge $e \in E(G)$ corresponds to an unordered pair $\{u, v\}$ of two vertices in $V(G)$, we write $e = uv$ or $e = vu$; and we say that *e* ***joins*** *u* and *v*; and we also say that *u* and *v* are ***adjacent***; *e* is ***incident*** with *u* and *v*; and the vertices *u* and *v* are called the ***ends*** of *e*. An edge with identical ends is called a ***loop*** and an edge with distinct ends is a ***link***. If two edges *e* and *f* join the same pair of vertices, then *e* and *f* are called ***parallel edges***. A graph is said to be ***simple*** if it contains no loops and no parallel edges. Throughout this paper we will consider only simple graphs. A simple graph in which each pair of distinct vertices is joined by an edge is called a ***complete graph***. A complete graph on *n* vertices is denoted by K_n. A ***walk*** in a graph *G* is a finite sequence $W = v_0 e_1 v_1 e_2 v_2 \cdots e_k v_k$ whose terms are alternately vertices and edges, such that, for $1 \le i \le k$, the ends of e_i are v_{i-1} and v_i.The vertices v_0 and v_k are called the ***origin*** and ***terminus*** of *W* respectively, and $v_1, v_2, \cdots, v_{k-1}$, its ***internal vertices***. If it does not lead to confusion, we will simply denote the walk *W* by the sequence $v_0 v_1 v_2 \cdots v_k$ of its vertices. The ***length*** of a walk is the number of edges appearing in it, and so the walk given above has length *k*. If the edges $e_1, e_2, \cdots, e_k$ of a walk *W* are distinct, *W* is called a ***trail***. If, in addition, $v_0, v_1, v_2, \cdots, v_k$ are distinct, *W* is called a ***path***. A walk (respectively a path) with origin *u* and terminus *v* is called a (*u*, *v*)-***walk*** (respectively a (*u*, *v*)-***path***). A graph *G* is called ***connected*** if for any two vertices *u* and *v* in *G* there is a (*u*, *v*)-path, otherwise *G* is ***disconnected***. A ***subgraph*** of a graph $G=(V(G),E(G))$ is a graph $H=(V(H),E(H))$ with $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and it is written by $H \subseteq G$. If $H \subseteq G$ but $H \ne G$, we write $H \subset G$ and it is called a ***proper subgraph*** of *G*. Suppose that V' is a nonempty subset of $V(G)$. The subgraph of G whose vertex set is V' and whose edge set is the set of those edges of G that have both ends in V' is called the subgraph of G ***induced*** by V' and is denoted by $G[V']$; and we say that $G[V']$ is an ***induced subgraph*** of G. A maximal connected subgraph of *G* is called a

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component of *G*. A walk $v_0 e_1 v_1 e_2 \cdots e_k v_k e_{k+1} v_0$ of a graph *G* is a ***cycle*** if $k \geq 2$ and all the vertices $v_0, v_1, v_2, \cdots, v_k$ are distinct. A connected graph without a cycle is called a ***tree***. A vertex *v* of a graph *G* is a ***cut vertex*** if $G-v$ has more components than *G* where $G-v$ means the graph obtained from *G* by deleting the vertex *v* and all edges incident with *v*. A connected graph without a cut vertex is called a ***block***. A ***block of a graph*** *G* is a subgraph of *G* which is a block and is maximal with respect to this property. An edge *e* of a graph *G* which is a ***cut edge*** if $G-e$ has more components than *G* where $G-e$ means the graph obtained from *G* by deleting the edge *e*.

2. The Vertex-to-Vertex Median of a Graph

In this section we give the definition of the vertex-to-vertex median of a graph and state known results on it.

2.1 Definitions. Let *G* be a connected graph with the vertex set $V(G)$ and the edge set $E(G)$. If *u* and *v* are two vertices in *G*, the ***vertex-to-vertex distance*** between *u* and *v* is denoted by $d(u,v)$ and defined as the length of a shortest path joining them. The ***vertex-to-vertex distance sum*** $s(v)$ of a vertex *v* of *G* is

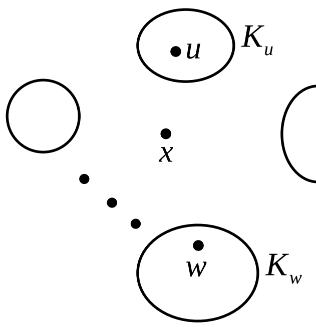
$$s(v) = \sum_{u \in V(G)} d(v, u).$$

The subgraph of G induced by the set of all vertices of G with minimum vertex-to-vertex distance sum is called the ***vertex-to-vertex median*** of G and is denoted by $M(G)$.

An interesting result on the structure of the vertex-to-vertex median of a connected graph is given below.

2.2 Theorem. If *G* is a connected graph, all vertices of the vertex-to-vertex median $M(G)$ of *G* lie in the same block of *G*.

Proof. To prove the theorem by contradiction, suppose that there exist two vertices *u* and *w* of $M(G)$ lying in distinct blocks of *G*. This implies that there exists a cut vertex *x* of *G* such that *u* and *w* lie in distinct components of $G-x$. Let K_u and K_w be the components of $G-x$ containing the vertex *u* and *w* respectively.



Suppose that

(1) $$|V(K_u)| < \frac{1}{2}|V(G)|.$$

It is obvious that

$d(x,v) \le d(x,u) + d(u,v)$ for each vertex v in K_u,

that is,

(2) $$d(x,v) \le d(u,v) + d(x,u)$$

and

$d(u,v) = d(u,x) + d(x,v)$ for every vertex v not in K_u,

that is,

(3) $$d(x,v) = d(u,v) - d(x,u).$$

Therefore

$$s(x) = \sum_{v \in V(G)} d(x,v)$$

$$= \sum_{v \in V(K_u)} d(x,v) + \sum_{v \notin V(K_u)} d(x,v)$$

and by using (2) and (3),

$$s(x) \le \sum_{v \in V(K_u)} (d(u,v) + d(x,u)) + \sum_{v \notin V(K_u)} (d(u,v) - d(x,u))$$

$$= \sum_{v \in V(G)} d(u,v) + d(x,u)\left[|V(K_u)| - (|V(G)| - |V(K_u)|)\right]$$

(4) $$= s(u) + d(x,u)\left[2|V(K_u)| - |V(G)|\right].$$

From (1) and (4), we obtain

$$s(x) < s(u)$$

and this is a contradiction to the fact that u is a vertex in $M(G)$. Therefore our assumption (1) is false and hence

(5) $$|V(K_u)| \ge \frac{1}{2}|V(G)|.$$

Similarly,

(6) $$|V(K_w)| \ge \frac{1}{2}|V(G)|.$$

By (5) and (6),

$$|V(K_u)| + |V(K_w)| \ge |V(G)|$$

and this is impossible since

$$V(K_u) \cup V(K_w) \cup \{x\} \subseteq V(G).$$

Thus there do not exist two vertices *u* and *w* of *M*(*G*) lying in distinct blocks of *G*, and this means that all vertices of *M*(*G*) lie in a block of *G*.

2.3 Corollary. If *T* is a tree, then the vertex set of the vertex-to-vertex median *M*(*T*) of *T* consists of one vertex or two adjacent vertices.

Proof. The corollary easily follows from Theorem 2.2 and the fact that each block of a tree *T* is an edge.

2.4 Definitions. A connected graph containing exactly one cycle is called a ***unicyclic graph***. A connected graph containing exactly two cycles is called a ***bicyclic graph***.

2.5 Corollary. Let *G* be a unicyclic graph containing a cycle *C*. Then the vertex set of the vertex-to-vertex median *M*(*G*) of *G* consists of one vertex or two adjacent vertices or some vertices of the cycle *C*.

Proof. The corollary easily follows from Theorem 2.2 and the fact that a block of the unicyclic graph *G* is either an edge (not in *C*) or the cycle *C*.

2.6 Corollary. Let *G* be a bicyclic graph containing two cycles C_1 and C_2. Then the vertex set of the vertex-to-vertex median *M*(*G*) of *G* consists of one vertex or two adjacent vertices or some vertices of C_1 or some vertices of C_2.

Proof. The corollary easily follows from Theorem 2.2 and the fact that a block of the bicyclic graph *G* is an edge (not in C_1 or C_2) or the cycle C_1 or the cycle C_2.

3. The Vertex-to-Edge Median of a Graph

In this section we describe the concept of the vertex-to-edge median of a graph introduced by Santhakumaran.

3.1 Definitions. Let *G* be a connected graph with the vertex set $V(G)$ and the edge set $E(G)$. If *v* is a vertex and $f = xy$ is an edge of *G*, the ***vertex-to-edge distance*** between *v* and *f* is denoted by $d(v, f)$ and defined as

$$d(v, f) = \min\{d(v, x),\ d(v, y)\}.$$

The ***vertex-to-edge distance sum*** $s_1(v)$ of a vertex *v* in *G* is

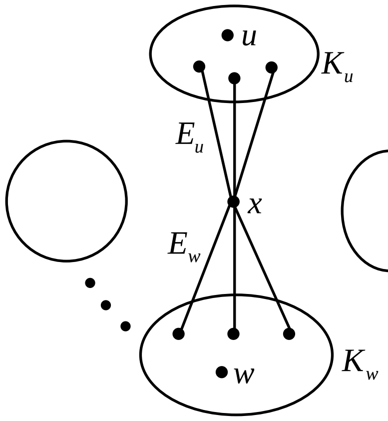
$$s_1(v) = \sum_{e \in E(G)} d(v, e).$$

The subgraph of *G* induced by the set of all vertices of *G* with minimum vertex-to-edge distance sum is called the ***vertex-to-edge median*** of *G* and is denoted by $M_1(G)$.

The next theorem on the structure of the vertex-to-edge median of a connected graph is a counter path of Theorem 2.2.

3.2 Theorem. If *G* is a connected graph, all vertices of the vertex-to-edge median $M_1(G)$ of *G* lie in the same block of *G*.

Proof. To prove the theorem by contradiction, suppose that there exist two vertices *u* and *w* of $M_1(G)$ lying in distinct blocks of *G*. This implies that there exists a cut vertex *x* of *G* such that *u* and *w* lie in distinct components of $G-x$.



Let K_u and K_w be the components of $G-x$ containing the vertex *u* and *w* respectively.

Let the edge sets E_u and E_w be defined by

$$E_u = \{ xy : y \in V(K_u)\},$$

$$E_w = \{ xy : y \in V(K_w)\}.$$

We also define the edge sets E'_u and E'_w as follows:

$$E'_u = E_u \bigcup E(K_u),$$

$$E'_w = E_w \bigcup E(K_w).$$

Suppose that

(1) $\quad |E'_u| \le \frac{1}{2}|E(G)|.$

It is easy to see that

(2) $\quad d(x,e) \le d(x,u) + d(u,e)$ for each edge *e* in $E(K_u)$,

(3) $\quad d(x,e) < d(x,u) + d(u,e)$ since $d(x,e) = 0$,

and

$$d(u,e) = d(u,x) + d(x,e) \text{ for each edge } e \text{ in } E(G) \setminus (E(K_u) \bigcup E_u) = E(G) \setminus E'_u,$$

that is,

(4) $\quad d(x,e) = d(u,e) - d(x,u).$

Now

$$s_1(x) = \sum_{e \in E(G)} d(x,e)$$

$$= \sum_{e \in E'_u} d(x,e) + \sum_{e \in E(G) \setminus E'_u} d(x,e)$$

and by using (2), (3) and (4), we get

$$s_1(x) < \sum_{e \in E'_u} \left[d(x,u) + d(u,e) \right] + \sum_{e \in E(G) \setminus E'_u} \left[d(u,e) - d(x,u) \right]$$

$$= \sum_{e \in E(G)} d(u,e) + d(x,u) \left[|E'_u| - |E(G) \setminus E'_u| \right]$$

$$= s_1(u) + d(x,u) \left[2|E'_u| - |E(G)| \right]$$

$$\le s_1(u)$$

by virtue of our assumption (1) and we have a contradiction to the fact that *u* is a vertex of $M_1(G)$. Therefore our assumption (1) must be false and we must have

(5) $$|E'_u| > \frac{1}{2}|E(G)|.$$

Similarly, we have

(6) $$|E'_w| > \frac{1}{2}|E(G)|$$

and by combining (5) and (6), we obtain

$$|E'_u| + |E'_w| > |E(G)|.$$

This is impossible since

$$E'_u \cap E'_w = \varnothing \text{ and } E'_u \cup E'_w \subseteq E(G).$$

So the vertices *u* and *w* cannot lie in distinct blocks of *G*. This means that all vertices of $M_1(G)$ lie in the same block of *G* and the theorem is proved.

3.3 Corollary. If *T* is a tree, then the vertex set of the vertex-to-edge median $M_1(T)$ of *T* consists of one vertex or two adjacent vertices.

Proof. The corollary easily follows from Theorem 3.2 and the fact that each block of a tree *T* is an edge.

3.4 Corollary. Let *G* be a unicyclic graph containing a cycle *C*. Then the vertex set of the vertex-to-edge median $M_1(G)$ of *G* consists of one vertex or two adjacent vertices or some vertices of the cycle *C*.

Proof. The corollary easily follows from Theorem 3.2 and the fact that a block of the unicyclic graph *G* is either an edge (not in *C*) or the cycle *C* in *G*.

3.5 Corollary. If G is a bicyclic graph containing exactly two cycles C_1 and C_2, then the vertex set of the vertex-to-edge median $M_1(G)$ of G consists of one vertex or two adjacent vertices or some vertices of C_1 or some vertices of C_2.

Proof. The corollary easily follows from Theorem 3.2 and the fact that a block of the bicyclic graph G is an edge (not in C_1 or C_2) or the cycle C_1 or the cycle C_2.

4. Vertex-to-Vertex Median and Vertex-to-Edge Median of a Double Lollipop

In this section, we investigate the structures of the vertex-to-vertex median and the vertex-to-edge median of a double lollipop which is a particular type of a bicyclic graph.

4.1 Definition. Let G be a connected graph with the vertex set $V(G)=\{x_1, x_2, x_3, \ldots, x_{m_1}, z_1, z_2, z_3, \ldots, z_n, y_1, y_2, y_3, ..., y_{m_2}\}$ and the edge set $E(G)=\{x_1x_2, x_2x_3, x_3x_4, \ldots, x_{m_1}x_1, x_1z_1, z_1z_2, \ldots, z_{n-1}z_n, z_ny_1, y_1y_2, y_2y_3, \ldots, y_{m_2}y_1\}$, where m_1, m_2 and n are positive integers with $m_1 \geq 3$, $m_2 \geq 3$ and $n \geq 1$, which consists of two cycles $C_1 = x_1 x_2 \ldots x_{m_1} x_1$, $C_2 = y_1 y_2 \ldots y_{m_2} y_1$ and a path $P = x_1 z_1 z_2 \ldots z_n y_1$. Then G is called a ***double lollipop***, see Fig. 1.

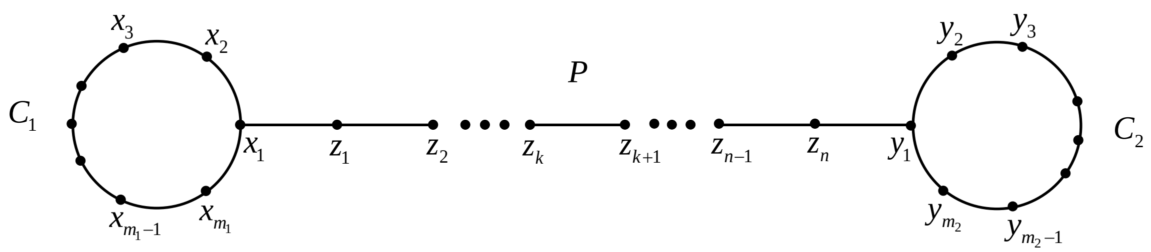


Figure 1

In investigating the structure of the vertex-to-vertex median and the vertex-to-edge median of a double lollipop, the following two propositions and two lemmas are useful.

4.2 Proposition. Let G be a connected graph, xy be a cut edge of G, and G_x and G_y be the components of $G - xy$ containing the vertex x and y respectively. Then

(1) $s(x) < s(y) \Leftrightarrow |V(G_x)| > |V(G_y)|$,

(2) $s(x) = s(y) \Leftrightarrow |V(G_x)| = |V(G_y)|$.

Proof. We observe that

$$d(x, v) = d(y,v) - 1 \text{ for each } v \in V(G_x)$$

and

$$d(x, v) = d(y,v) + 1 \text{ for each } v \in V(G_y).$$

Therefore

$$s(x) = \sum_{v \in V(G)} d(x,v)$$

$$= \sum_{v \in V(G_x)} d(x,v) + \sum_{v \in V(G_y)} d(x,v)$$

$$= \sum_{v \in V(G_x)} (d(y,v) - 1) + \sum_{v \in V(G_y)} (d(y,v) + 1)$$

$$= \sum_{v \in V(G_x)} d(y,v) - |V(G_x)| + \sum_{v \in V(G_y)} d(y,v) + |V(G_y)|$$

$$= \sum_{v \in V(G)} d(y,v) + |V(G_y)| - |V(G_x)|.$$

So

(3) $s(x) = s(y) + |V(G_y)| - |V(G_x)|.$

Now it immediately follows from Equation (3) that

$$s(x) < s(y) \Leftrightarrow |V(G_x)| > |V(G_y)|$$

and

$$s(x) = s(y) \Leftrightarrow |V(G_x)| = |V(G_y)|.$$

4.3 Proposition. Let *G* be a connected graph, $e^* = xy$ be a cut edge of *G*, and G_x and G_y be the components of $G - xy$ containing the vertex *x* and *y* respectively. Then

(1) $s_1(x) < s_1(y) \Leftrightarrow |E(G_x)| > |E(G_y)|$,

(2) $s_1(x) = s_1(y) \Leftrightarrow |E(G_x)| = |E(G_y)|$.

Proof. We obverse that

$$d(x,e) = d(y,e) - 1 \text{ for each } e \in E(G_x),$$

$$d(x,e) = d(y,e) + 1 \text{ for each } e \in E(G_y)$$

and

$$d(x, e^*) = 0 = d(y, e^*).$$

Therefore

$$s_1(x) = \sum_{e \in E(G)} d(x,e)$$

$$= d(x,e^*) + \sum_{e \in E(G_x)} d(x,e) + \sum_{e \in E(G_y)} d(x,e)$$

$$= d(y,e^*) + \sum_{e \in E(G_x)} (d(y,e) - 1) + \sum_{e \in E(G_y)} (d(y,e) + 1)$$

$$= d(y,e^*) + \sum_{e\in E(G_x)} d(y,e) - |E(G_x)| + \sum_{e\in E(G_y)} d(y,e) + |E(G_y)|.$$

So

(3) $s_1(x) = s_1(y) - |E(G_x)| + |E(G_y)|$.

Now it immediately follows from Equation (3) that

$$s_1(x) < s_1(y) \Leftrightarrow |E(G_x)| > |E(G_y)|$$

and

$$s_1(x) = s_1(y) \Leftrightarrow |E(G_x)| = |E(G_y)|.$$

4.4 Lemma. Let G be a double lollipop given in Definition 4.1. For any i with $2 \le i \le m_1$

(1) $s(x_i) > s(x_1)$,

(2) $s_1(x_i) > s_1(x_1)$.

Proof. (1) It is not difficult to see that

$$\sum_{j=1}^{m_1} d(x_i, x_j) = \sum_{j=1}^{m_1} d(x_1, x_j)$$

and that

$$d(x_i, v) > d(x_1, v)$$

for any vertex $v \in V(G) \setminus V(C_1)$.

It follows from the above equation and inequality that

$$s(x_i) > s(x_1).$$

(2) It is clear that

$$\sum_{e\in E(C_1)} d(x_i, e) = \sum_{e\in E(C_1)} d(x_1, e)$$

and that

$$d(x_i, e) > d(x_1, e)$$

for any edge $e \in E(G) \setminus \mathrm{E}(C_1)$.

It follows from the above equation and inequality that

$$s_1(x_i) > s_1(x_1).$$

Similarly we can prove the following lemma.

4.5 Lemma. Let G be a double lollipop given in Definition 4.1. For any i with $2 \le i \le m_2$

(1) $s(y_i) > s(y_1)$,

(2) $s_1(y_i) > s_1(y_1)$.

By using the above propositions and lemmas, we can now derive some results on the vertex-to-vertex median and the vertex-to-edge median of a double lollipop.

4.6 Theorem. Let G be a double lollipop given in Definition 4.1. Suppose that $m_1 > m_2 + n$. Then $V(M(G)) = V(M_1(G)) = \{x_1\}$.

Proof. Since $m_1 > m_2 + n$, it follows from Propositions 4.2 and 4.3 that

$$s(x_1) < s(z_1) < s(z_2) < \cdots < s(z_n) < s(y_1),$$

$$s_1(x_1) < s_1(z_1) < s_1(z_2) < \cdots < s_1(z_n) < s_1(y_1).$$

From these inequalities, Lemma 4.4 and Lemma 4.5, it follows that

$$V(M(G)) = V(M_1(G)) = \{x_1\}.$$

Similarly we can prove the next theorem.

4.7 Theorem. Let G be a double lollipop given in Definition 4.1. Suppose that $m_2 > m_1 + n$. Then $V(M(G)) = V(M_1(G)) = \{y_1\}$.

4.8 Theorem. Let G be a double lollipop given in Definition 4.1. Suppose that $m_1 = m_2 + n$. Then $V(M(G)) = V(M_1(G)) = \{x_1, z_1\}$.

Proof. Since $m_1 = m_2 + n$, it follows from Propositions 4.2 and 4.3 that

$$s(x_1) = s(z_1) < s(z_2) < \cdots < s(z_n) < s(y_1),$$

$$s_1(x_1) = s_1(z_1) < s_1(z_2) < \cdots < s_1(z_n) < s_1(y_1).$$

From these inequalities, Lemma 4.4 and Lemma 4.5, it follows that

$$V(M(G)) = V(M_1(G)) = \{x_1, z_1\}.$$

Similarly we can prove the following theorem.

4.9 Theorem. Let G be a double lollipop given in Definition 4.1. Suppose that $m_2 = m_1 + n$. Then $V(M(G)) = V(M_1(G)) = \{z_n, y_1\}$.

4.10 Theorem. Let G be a double lollipop given in Definition 4.1. Let $m_1 < m_2 + n$, $m_2 < m_1 + n$ and $m_1 + m_2 + n$ is even. Then

$$V(M(G)) = V(M_1(G)) = \{z_k, z_{k+1}\}$$

where $$k = \frac{m_1 + m_2 + n}{2} - m_1.$$

Proof. By our choice of *k* we have

$$\left|V(G_{z_k})\right| = \left|V(G_{z_{k+1}})\right| = \frac{m_1 + m_2 + n}{2},$$

$$\left|E(G_{z_k})\right| = \left|E(G_{z_{k+1}})\right| = \frac{m_1 + m_2 + n}{2}.$$

Therefore by Propositions 4.2 and 4.3, we obtain

$$s(x_1) > s(z_1) > s(z_2) > \cdots > s(z_k) = s(z_{k+1}) < s(z_{k+2}) < \cdots < s(z_n) < s(y_1)$$

and

$$s_1(x_1) > s_1(z_1) > s_1(z_2) > \cdots > s_1(z_k) = s_1(z_{k+1}) < s_1(z_{k+2}) < \cdots < s_1(z_n) < s_1(y_1).$$

From these relations, Lemma 4.4 and Lemma 4.5, it follows that

$$V(M(G)) = V(M_1(G)) = \{z_k, z_{k+1}\}.$$

4.11 Theorem. Let G be a double lollipop given in Definition 4.1. Suppose that $m_1 < m_2 + n$, $m_2 < m_1 + n$ and $m_1 + m_2 + n$ is odd. Then

$$V(M(G)) = V(M_1(G)) = \{z_k\}$$

where $$k = \frac{m_1 + m_2 + n + 1}{2} - m_1.$$

Proof. From our choice of *k* and Propositions 4.2 and 4.3, it follows that

$$s(x_1) > s(z_1) > s(z_2) > \cdots > s(z_{k-1}) > s(z_k) < s(z_{k+1}) < s(z_{k+2}) < \cdots < s(z_n) < s(y_1)$$

and

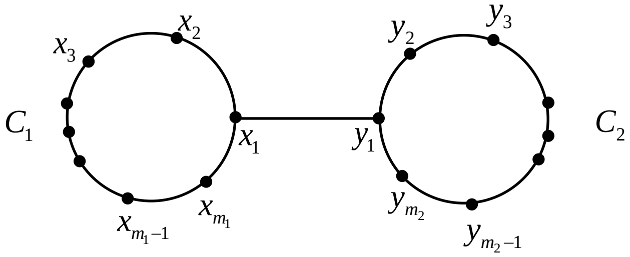
$$s_1(x_1) > s_1(z_1) > s_1(z_2) > \cdots > s_1(z_{k-1}) > s_1(z_k) < s_1(z_{k+1}) < s_1(z_{k+2})$$
$$< \cdots < s_1(z_n) < s_1(y_1).$$

From these relations, Lemma 4.4 and Lemma 4.5, imply that

$$V(M(G)) = V(M_1(G)) = \{z_k\}.$$

This completes the investigation of the structures of the vertex-to-vertex medians and vertex-to-edge medians of double lollipops.

4.12 Remark. Our definition of a double lollipop does not include the following graph.



However, by using similar arguments we have applied above we can prove the following.

(1) If $m_1 > m_2$, then

$$V(M(G)) = V(M_1(G)) = \{x_1\}.$$

(2) If $m_2 > m_1$, then

$$V(M(G)) = V(M_1(G)) = \{y_1\}.$$

(3) If $m_1 = m_2$, then

$$V(M(G)) = V(M_1(G)) = \{x_1, y_1\}.$$

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